**Math 120  
4.3 Properties of Logarithms**

# Objectives:

1. Use the product rule.
2. Use the quotient rule
3. Use the power rule.
4. Expand logarithmic expressions.
5. Condense logarithmic expressions.
6. Use the change-of-base property.

# Topic #1: Properties of Logarithms – The Product Rule

Since logarithms are exponents in reverse, certain properties of exponents apply to logarithms too. Recall the definition of logarithm:



The product rule for exponents states:



This indicates that to **multiply** like base exponents means to **add** the powers.



The Product Rule for logarithms is similar:



This also indicates that multiplication becomes addition.

For example, we can expand a logarithmic expression that involves multiplication:



*Example #1* – Expand the Expression and Simplify when Possible

a)



The product of and is an input to the operation “log base 6” and the product rule applies:

b)



The product of and is an input to the operation “log base 10” and the product rule applies:



The first term of the expansion can be simplified since :



c)

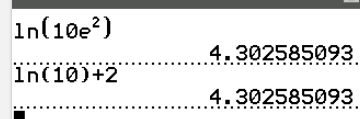


The product of and is an input to the operation “log base ” and the product rule applies



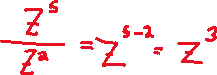
The second term of the expansion can be simplified since a natural log (base ) undoes a base exponent:





# Topic #2: Properties of Logarithms – The Quotient Rule

The quotient rule for exponents states:



This indicates that to **divide** like base exponents means to **subtract** the powers (in order).

The corresponding Quotient Rule for logarithms is similar:



This also indicates that division becomes subtraction.

For example, we can expand a logarithmic expression that involves division:



*Example #1* – Expand the Expression and Simplify when Possible



The quotient of and is an input to the operation “log base 9” and the quotient rule applies. The first term of the expansion can be simplified since :



The quotient of and is an input to the operation to the common log (base 9) and the quotient rule applies. The second term of the expansion can be simplified since :



The quotient of and is an input to the operation to the natural log (base ) and the quotient rule applies. The first term of the expansion can be simplified since the natural log undoes base :





# Topic #3: Properties of Logarithms – The Power Rule

The power rule for exponents states:



This indicates that to raise a **power to a power** is to **multiply** the powers.

The Power Rule for logarithms is similar:



This also indicates that raising a power becomes multiplication.

For example, we can rewrite a logarithmic expression with a power to become one with multiplication:

*Example #1* – Expand the Expression

a)



The base to the power of is an input to the operation “log base ” (which can be any suitable base); the power rule applies and the power of becomes a multiplier/coefficient:

b)



The fractional power of square root is “one half”. The base to the power of is an input to the operation of the natural log; the power rule applies and the power of becomes a multiplier/coefficient:

c)



The base to the power of is an input to the operation of the common log; the power rule applies and the power of becomes a multiplier/coefficient:



# Topic #4: Properties of Logarithms – Using Multiple Rules

Some logarithmic expressions can be expanded by using multiple rules. Here are the three fundamental properties again:

Product Rule:



Quotient Rule:



Power Rule:



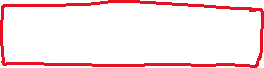
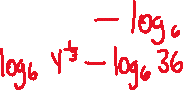
*Example #1* – Expand the Expression and Simplify when Possible



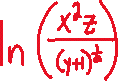
Rewrite the radical as a fractional power; there is a product rule and two power rules:



Rewrite the radical as a fractional power; there is a quotient rule, a product rule in the denominator, and two power rules:



Rewrite the radical as a fractional power; there is a quotient rule, a product rule in the numerator, and two power rules:

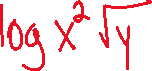
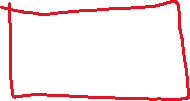


# Topic #5: Properties of Logarithms in Reverse – Condensing a Logarithmic Expression

The rules established above are a two-way street. We can expand a logarithmic expression into two or more terms, but we can also condense two or more terms into one expression. Here are the rules (again) in reverse:

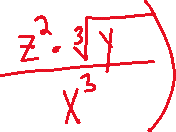
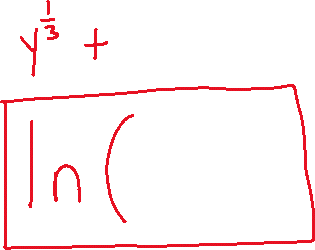
Consider the expression

We have two terms; the coefficients become powers (the half power can be written as a square root) and addition becomes multiplication:

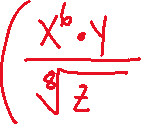


*Example #1* – Condense the Expression

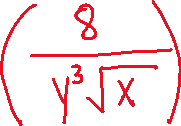
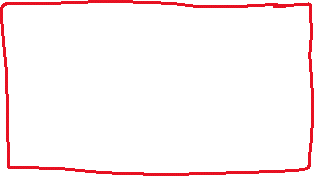
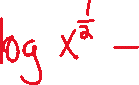
Rewrite the coefficients as powers; positive terms make up the numerator and negative terms make up the denominator:



Rewrite the coefficients as powers (the coefficient for the third term is , so we do not need to do anything); positive terms make up the numerator and negative terms make up the denominator:



Rewrite the coefficients as powers (the coefficient for the first term is , so we do not need to do anything); positive terms make up the numerator and negative terms make up the denominator:



# Topic #6: Properties of Logarithms – Change of Base

Not all values of logarithmic expressions are rational numbers. For example is not a rational number, it is somewhere between and since and

Scientific and graphing calculators are programmed to give values for common logs (base 10):



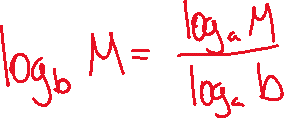
Scientific and graphing calculators also evaluate natural logs (base ). Using a calculator, we can approximate values such as:



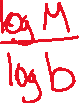
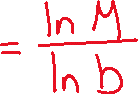
Most calculators do not directly evaluate other bases. For example is not a rational number, it is somewhere between and 4 since and . However, most calculators do not give this value directly.



The Change of Base Property is the way around this issue:



The property tell us we can express the logarithm of base in terms of another base . The two most convenient and useful bases are base and base :

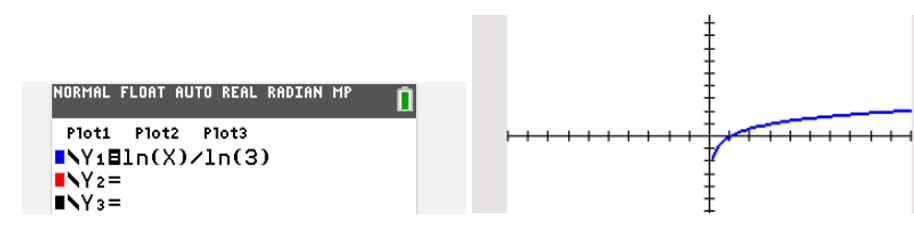


We can use either common logs or natural logs to evaluate other bases. For example,

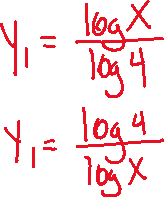


We can also graph logarithmic functions other bases with this property. Suppose we want to graph .

Using change of base (pick either a common or natural base):



*Example #1* – Graph the following in your calculator.



*Example #2* – Find the following in your calculator. Round to the nearest hundredth.

